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Guessing and converging periodic orbits in fluid flows with machine learning

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Abstract

Unstable periodic orbits (UPOs) are the underlying dynamical structures of chaotic ODEs. Since spatio-temporally chaotic PDEs are usually discretized, they are numerically represented by a large set of coupled ODEs. Together with the identification of UPOs (Kawahara and Kida, JFM, 2001) in the Navier-Stokes equations, this begs the question if a 'periodic orbit theory' is also applicable to chaotic fluid flows. While finding these UPOs is highly challenging, it is usually done in two steps: defining an initial guess and converging this guess to machine precision. The high-dimensionality of the discretized state space of fluid flows makes both of these steps difficult.

With machine learning proving very efficient in nonlinear dimensionality reduction, we present a data-driven method for generating guesses that exploits the collapse of the dissipative dynamics onto a chaotic attractor embedded in a low-dimensional curved manifold. We use autoencoders to construct an approximation of the coordinates for this manifold, and leverage this reduction in dimensionality to construct initial guesses for periodic orbits. In the low-dimensional latent space, we define guesses for UPOs using two methods: first by randomly sampling closed curves based on the latent POD modes which match moments of the latent flow statistics, and secondly by 'gluing' known UPOs at their closest point of passage. These 'loops' are decoded to physical space and used as initial guesses in variational convergence algorithms (Azimi et al., PRE, 2022). These guesses prove to be successful, and in particular the 'gluing' method results in longer UPOs, appealing to a hierarchy where long UPOs are concatenations of shorter ones. Finally, we discuss recent progress in defining the manifold dynamics. Having such a dynamics allows for convergence algorithms to be implemented directly in the latent space, resulting in an optimization process with fewer variables by multiple orders of magnitude.

Keywords:

Unstable periodic orbits, chaos, Navier-Stokes, autoencoders

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